Get Ready!

- Squaring Numbers (Simplify.)
  \[3^2 \quad 4^2 \quad 11^2\]

- Simplifying Expressions (Simplify each expression. Use 3.14 for \(\pi\).)
  \[2 \cdot 7.5 + 2 \cdot 11 \quad \pi(5)^2 \quad \sqrt{5^2 + 12^2}\]

- Evaluating Expressions (Evaluate the following expressions for \(a = 4\) and \(b = -2\).)
  \[\frac{a+b}{2} \quad \frac{a-7}{3-b} \quad \sqrt{(7 - a)^2 + (2 - b)^2}\]
Get Ready!

• Finding Absolute Value (Simplify each absolute value expression.)
  \[ |−8| \quad |2 − 6| \quad |−5 − (−8)| \]

• Solving Equations (Solve each equation.)
  \[ 2x + 7 = 13 \quad 5x − 12 = 2x + 6 \quad 2(x + 3) − 1 = 7x \]
Looking Ahead Vocabulary

• A child can *construct* models of buildings by stacking and arranging colored blocks. What might the term *construction* mean in geometry?

• The *Mid-Autumn Festival*, celebrated in China, falls exactly in the middle of autumn, according to the Chinese lunar calendar. What would you expect a *midpoint* to be in geometry?

• Artists often use long streaks to show *rays* of light coming from the sun. A ray is also a geometric figure. What do you think the properties of a *ray* are?

• You and your friend work with each other. In other words, you and your friend are co-workers. What might the term *collinear* mean in geometry?
Tools of Geometry

Unit 1
Nets and Drawings for Visualizing Geometry

Objective: To make nets and drawings of three-dimensional figures.
Objectives

• I can identify a solid form a net.
• I can draw a new from a solid.
• I can make an isometric drawing.
• I can make an orthographic drawing.
Vocabulary

• Visualizing figures is a key skill that you will develop in geometry.

• You can represent a three-dimensional object with a two-dimensional figure using special drawing techniques.

• A net is a two-dimensional diagram that you can fold to form a three-dimensional figure. A net shows all of the surfaces of a figure in one view.
Identifying a Solid From a Net

- The net folds into the cube shown beside it. Which letters will be on the top and front of the cube?
Practice

Match each three-dimensional figure with its net.

1. ___
2. ___
3. ___
4. ___
5. ___
6. ___
7. ___
Drawing a Net From a Solid

• What is a net for graham cracker box? Label the net with its dimensions.

• What is the net for the figure? Label the net with its dimensions.
Practice

• Draw a net for each figure. Label the net with its dimensions.

![Diagram of a prism with dimensions 4 in, 2 in, and 2 in.]

![Diagram of a cylinder with dimensions 3 m and 8 m.]
Vocabulary

• An **isometric drawing** shows a corner view of a three-dimensional figure. It allows you to see the top, front, and side of the figure. You can draw an isometric drawing on isometric dot paper.

• A net shows a three-dimensional figure as a folded-out flat surface. An isometric drawing shows a three-dimensional figure using slanted lines to represent depth.
Isometric Drawing

• What is an isometric drawing of each structure?
Practice

Make an isometric drawing of each cube structure on isometric dot paper.
Vocabulary

• An **orthographic drawing** is another way to represent a three-dimensional figure. An orthographic drawing shows three separate views: a top view, a front view, and a right-side view.

• Although an orthographic drawing may take more time to analyze, it provides unique information about the shape of a structure.
Orthographic Drawing

• What is the orthographic drawing for the isometric drawing?
Practice

• For each isometric drawing, make an orthographic drawing. Assume there are no hidden cubes.
Points, Lines, and Planes & Segment Length and Midpoints

Objective: To understand basic terms and postulates of geometry. To understand how to draw and measure a segment. To find the midpoint of a segment. To find and compare lengths of segments.
Objectives

• I can name points, lines, and planes.
• I can name segments and rays.
• I can find the intersection of two planes.
• I can construct a copy of a line segment.
• I can use the distance formula on the coordinate plane.
• I can find a midpoint.
• I can find a midpoint on the coordinate plane
Vocabulary

• Geometry is a mathematical system built on accepted facts, basic terms, and definitions.

• In geometry, some words such as point, line, and plane are undefined.

• Undefined terms are basic ideas that you can use to build the definitions of all other figures in geometry.

• Although you cannot define undefined terms, it is important to have a general description of their meanings.
<table>
<thead>
<tr>
<th>Term</th>
<th>Figure/Diagram</th>
<th>Ways to Name it</th>
</tr>
</thead>
<tbody>
<tr>
<td>A <strong>point</strong> indicates a location and has no size. It is a specific location, has no dimension, and is represented by a dot.</td>
<td><img src="image" alt="Point Diagram" /></td>
<td>You can represent a point by a dot and name it by a capital letter. Example: point $P$</td>
</tr>
<tr>
<td>A <strong>line</strong> is represented by a straight path that extends in two opposite directions without end and has no thickness. A line contains infinitely many points.</td>
<td><img src="image" alt="Line Diagram" /></td>
<td>You can name a line by any two points on the line, such as $\overline{AB}$ (read “line AB”) or $\overline{BA}$, or by a single lowercase letter, such as $\ell$.</td>
</tr>
<tr>
<td>A <strong>plane</strong> is represented by a flat surface that extends without end and has no thickness. A plane contains infinitely many lines.</td>
<td><img src="image" alt="Plane Diagram" /></td>
<td>You can name a plane by a capital letter, such as plane $P$, or by at least three points in the plane that do not all line on the same line, such as plane $ABC$.</td>
</tr>
</tbody>
</table>
Vocabulary

• Points that lie on the same line are collinear points.

• Points and lines that lie in the same plane are coplanar.

• All the points of a line are coplanar.
Naming Points, Lines, and Planes

- What are two other ways to name plane
- What are two other ways to name plane
- What are the names of three collinear points?
- What are the names of four
Practice

What are two other ways to name plane V?

What are two lines?

Name three coplanar points.

Name two collinear points.
Practice

What are two lines?

Name three coplanar points.

Name two collinear points.
Vocabulary

• The terms point, line, and plane are not defined because their definitions would require terms that also need defining.

• You can, however, use undefined terms to define other terms.

• A geometric figure is a set of points.

• Space is the set of all points in three dimensions.

• Similarly, the definitions of segment and ray are based on points and lines.
## Vocabulary

<table>
<thead>
<tr>
<th>Term</th>
<th>Figure/Diagram</th>
<th>Ways to Name it</th>
</tr>
</thead>
<tbody>
<tr>
<td>A segment is part of a line that consists of two endpoints and all points between them.</td>
<td><img src="image" alt="Segment Diagram" /></td>
<td>You can name a segment by its two endpoints, such as $\overline{AB}$ (read “segment AB”) or $\overline{BA}$.</td>
</tr>
<tr>
<td>A ray is part of a line that consists of one endpoint and all the points of the line on one side of the endpoint.</td>
<td><img src="image" alt="Ray Diagram" /></td>
<td>You can name a ray by its endpoint and another point on the ray, such as $\overline{AB}$ (read “ray AB”). The order of points indicates the ray’s direction.</td>
</tr>
<tr>
<td>Opposite rays are two rays that share the same endpoint and form a line.</td>
<td><img src="image" alt="Opposite Rays Diagram" /></td>
<td>You can name opposite rays by their shared endpoint and any other point on each ray, such as $\overline{CA}$ and $\overline{CB}$.</td>
</tr>
</tbody>
</table>
Naming Segments and Rays

• What are the names of the segments in the figure?

• What are the names of the rays in the figure?

• Which of they rays are opposite rays?
Practice

• Name the segments in the figure.

• Name the rays in the figure.

• Name the pair of opposite rays with an endpoint T.

• Name another pair of opposite rays.
Vocabulary

• A **postulate** or **axiom** is an accepted statement of fact.

• Postulate, like undefined terms, are basic building blocks of the logical system in geometry.

• You will use logical reasoning to prove general concepts in this book.

• Postulate 1-1: Through any two points there is exactly one line. Line $t$ passes through points A and B. Line $t$ is the only line that passes through both points.
Vocabulary

• When you have two or more geometric figures, their intersection is the set of points the figures have in common.

• Postulate 1-2: If two distinct lines intersect, then they intersect in exactly one point. $\overrightarrow{AE}$ and $\overrightarrow{DB}$ intersect in point C.

• Postulate 1-3: If two distinct planes intersect, then they intersect in exactly one line. Plane QBA and Plane PAB intersect in $\overline{AB}$.

• When you know two points that two planes have in common, Postulates 1-1 and 1-3 tell you that the line through those points is the intersection of the planes.
Finding the Intersection of Two Planes

• What is the intersection of planes A and B?

• What is the intersection of planes B and C?

• What is the intersection of planes A and C?
Practice

• Name the intersection of each pair of planes.
  - Planes ABC and BCF
  - Planes AHE and HEF
  - Planes CDE and DEF
  - Planes GFC and BGF

• Name two planes that intersect in the given line.
  - $\overrightarrow{AD}$
  - $\overrightarrow{GH}$
  - $\overrightarrow{CF}$
  - $\overrightarrow{BG}$
Vocabulary

• When you name a plane from a figure like in the previous problem, list the corner points in consecutive order.

• Postulate 1-4: Through an three noncollinear points there is exactly one plane.
Using Postulate 1-4

• What plane contains points A, B, and F?

• What plane contains points G, H, and D?

• What plane contains points E, F, and H?

• Can you list any more planes?
Practice

• What plane contains points P, W, and Z?
• What plane contains points S, R, and Q?
• What plane contains points Y, X, and Q?
• Can you list any more planes?
Measuring Segments

Objective: To find and compare lengths of segments.
Objectives

• I can measure segment lengths.
• I can use the segment addition postulate.
• I can compare segment lengths.
• I can use the midpoint.
Vocabulary

• You can use number operations to find and compare the lengths of segments.

• Postulate 1-5 (Ruler Postulate): Every point on a line can be paired with a real number. This makes a one-to-one correspondence between the points on the line and the real numbers. The real number that corresponds to a point is called the coordinate of the point.

• The Ruler Postulate allows you to measure lengths of segments using a given unit and to find distances between points on a number line.
Volume

• The **distance** between points A and B is the absolute value of the difference of their coordinates, or \(|a - b|\).

• This value is also AB or the length of \(\overline{AB}\).

\[ AB = |a - b| \]
Measuring Segment Lengths

- What is ST?
- What is UV?
- What is SV?
- What is TU?
- What is SU?
- What is TV?
Practice

• Find the length of each segment.
  • $\overline{AD}$
  • $\overline{BE}$
  • $\overline{CD}$
  • $\overline{AB}$
  • $\overline{AE}$
  • $\overline{CE}$
  • $\overline{BD}$
  • $\overline{AC}$
  • $\overline{BC}$
  • $\overline{DE}$
Vocabulary

• Postulate 1-6 (Segment Addition Postulate): If three points A, B and C are collinear and B is between A and C, then $AB + BC = AC$. 
Using the Segment Addition Postulate

- If $EG = 59$, what are $EF$ and $FG$?

- In the diagram, $JL = 120$. What are $JK$ and $KL$?
Practice

• If RS = 15 and ST = 9, then RT = _____.

• If ST = 15 and RT = 40, then RS = _____.

• RS = 8y + 4, St = 4y + 8, and RT = 15y − 9.
  - What is the value of y?
  - Find RS, ST, and RT.
Vocabulary

• When numerical expressions have the same value, you say that they are equal (=).

• Similarly, if two segments have the same length, then the segments are congruents (≅) segments.

• This means that if AB = CD, then \( \overline{AB} \cong \overline{CD} \). You can also say that if \( \overline{AB} \cong \overline{CD} \), then AB = CD.
Comparing Segment Lengths

• Are $\overline{AC}$ and $\overline{BD}$ congruent?

• Is $\overline{AB}$ congruent to $\overline{DE}$?
Practice

Use the number line. Tell whether the segments are congruent.

- \( \overline{LN} \) and \( \overline{MQ} \)
- \( \overline{MP} \) and \( \overline{NQ} \)
- \( \overline{MN} \) and \( \overline{PQ} \)
- \( \overline{LP} \) and \( \overline{MQ} \)
Vocabulary

• The **midpoint** of a segment is a point that divides the segment into two congruent segments.

• A point, line, ray, or other segment that intersects a segment at its midpoint is said to *bisect* the segment.

• That point, line, ray, or segment is called a **segment bisector**.
Using the Midpoint

• Q is the midpoint of $\overline{PR}$. What are PQ, QR, and PR?

• U is the midpoint of $\overline{TV}$. What are TU, UV, and TV?
Practice

• A is the midpoint of $\overline{XY}$.
  • Find $XA$, $AY$, and $XY$.

• Find the value of $PT$.
  • $PT = 5x + 3$ and $TQ = 7x - 9$
  • $PT = 4x - 6$ and $TQ = 3x + 4$
  • $PT = 7x - 24$ and $TQ = 6x - 2$
Measuring Angles

Objective: To find and compare the measures of angles.
Objectives

• I can name angles.
• I can measure and classify angles.
• I can use congruent angles.
• I can use the angle addition postulate.
Vocabulary

• You can use number operations to find and compare the measure of angles.

• Angle Key Concept:

<table>
<thead>
<tr>
<th>Definition</th>
<th>How to Name It</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>An <strong>angle</strong> is formed by two rays with the same endpoint. &lt;br&gt; The rays are the <strong>sides</strong> of the angle. The endpoint is the <strong>vertex</strong> of the angle.</td>
<td>You can name an by &lt;br&gt; • Its vertex, ∠A &lt;br&gt; • A point on each ray and the vertex, ∠BAC or ∠CAB &lt;br&gt; • A number, ∠1</td>
<td><img src="" alt="Diagram" /> The sides of the angle are ( AB ) and ( AC ). The vertex is A.</td>
</tr>
</tbody>
</table>
Vocabulary

• When you name angles using three points, the vertex must go in the middle.

• The interior of an angle is the region containing all of the points between the two sides of the angle.

• The exterior of an angle is the region containing all of the points outside of the angle.
Naming Angles

What are two other names for each?

• \( \angle 1 \)
• \( \angle 2 \)
• \( \angle 3 \)
• \( \angle 4 \)
• \( \angle 5 \)
Practice

• Name each angle in 3 different ways.
Vocabulary

• One way to measure the size of an angle is in degrees.

• To indicate the measure of an angle, write a lowercase m in front of the angle symbol.

• A circle has 360°, so 1 degree is $\frac{1}{360}$ of a circle.

• A protractor forms half a circle and measures angles from 0° to 180°.
Vocabulary

• Postulate 1-7 (Protractor Postulate): Consider $\overrightarrow{OB}$ and a point $A$ on one side of $\overrightarrow{OB}$. Every ray of the form $\overrightarrow{OA}$ can be paired one to one with a real number from 0 to 180.

• The Protractor Postulate allows you to find the measure of an angle.
Vocabulary

• The **measure** of $\angle COD$ is the absolute value of the difference of the real numbers paired with $\overrightarrow{OC}$ and $\overrightarrow{OD}$. That is, if $\overrightarrow{OC}$ corresponds with $c$, and $\overrightarrow{OD}$ corresponds with $d$, then $m\angle COD = |c - d|$.

• Notice that the Protractor Postulate and the calculator of an angle measure are very similar to the Ruler Postulate and the calculation of a segment length.
Vocabulary

• Types of Angles:

4 Types of Angles

- **Acute Angle**: an angle whose measure is less than 90°.
- **Right Angle**: an angle whose measure is exactly 90°.
- **Obtuse Angle**: an angle whose measure is between 90° and 180°.
- **Straight Angle**: an angle that is exactly 180°.
Measuring and Classifying Angles

What are the measures of $\angle COB$, $\angle COD$, $\angle AOC$, $\angle AOD$, $\angle AOB$?
Practice

• Find the measure of each angle.
Vocabulary

• Angles with the same measure are congruent angles. This means that if $m \angle A = m \angle B$, then $\angle A \cong \angle B$. You can also say that if $\angle A \cong \angle B$, then $m \angle A = m \angle B$.

• You can mark angles with arcs to show that they are congruent. If there is more than one set of congruent angles, each set is marked with the same number of arcs.
Using Congruent Angles

• Synchronized swimmers form angles with their bodies. If \( \angle GHJ = 90 \), what is \( \angle KLM \)?

• If \( \angle ABC = 49 \), what is \( \angle DEF \)?
Practice

• Complete each statement.
  • $\angle CBJ = ___$
  • $\angle FJH = ___$
  • If $m\angle EFD = 75$, then $m\angle JAB = ___$
  • If $m\angle GHF = 130$, then $m\angle JBC = ___$
Vocabulary

• The Angle Addition Postulate is similar to the Segment Addition Postulate.

• Postulate 1-8 (Angle Addition Postulate): If point B is the interior of \( \angle AOC \) then \( m\angle AOB + m\angle BOC = m\angle AOC \).
Using the Angle Addition Postulate

• If $m\angle RQT = 155$, what are $m\angle RQS$ and $m\angle TQS$?

• $\angle DEF$ is a straight angle. What are $m\angle DEC$ and $m\angle CEF$?
Practice

• If $\angle ABD = 79$, what are $\angle ABC$ and $\angle DBC$?

• $\angle RQT$ is a straight angle. What are $\angle RQS$ and $\angle TQS$?
Exploring Angle Pairs

Objective: To identify special angle pairs and use their relationships to find angle measures.
Objectives

• I can identify angle pairs.
• I can make conclusions from a diagram
• I can find missing angle measures.
• I can use an angle bisector to find angle measures.
Vocabulary

• Special angle pairs can help you identify geometric relationships. You can use these angle pairs to find angle measures.

• Types of angles:

<table>
<thead>
<tr>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Adjacent angles</strong> are two coplanar angles with a common side, a common vertex, and non common interior points.</td>
<td>( \angle A ) and ( \angle B )</td>
</tr>
<tr>
<td><strong>Vertical angles</strong> are two angles whose sides are opposite rays.</td>
<td>( \angle 1 ) and ( \angle 3, \angle 2 ) and ( \angle 4 )</td>
</tr>
<tr>
<td><strong>Complementary angles</strong> are two angles have a sum of 90. Each angle is called the complement of the other.</td>
<td></td>
</tr>
<tr>
<td><strong>Supplementary angles</strong> are two angles whose measures have a sum of 180. Each angle is called the supplement of the other.</td>
<td></td>
</tr>
</tbody>
</table>
Identifying Angle Pairs

Use the diagram. Is the statement true? Explain.

1. \( \angle BFD \) and \( \angle CFD \) are adjacent angles.
2. \( \angle AFB \) and \( \angle EFD \) are vertical angles.
3. \( \angle AFE \) and \( \angle BFC \) are complementary.
4. \( \angle AFE \) and \( \angle CFD \) are vertical angles.
5. \( \angle BFC \) and \( \angle DFE \) are supplementary.
6. \( \angle BFD \) and \( \angle AFB \) are adjacent angles.
Practice

Use the diagram. Is each statement true? Explain.

1. $\angle 1$ and $\angle 5$ are adjacent angles.
2. $\angle 3$ and $\angle 5$ are vertical angles.
3. $\angle 3$ and $\angle 4$ are complementary.
4. $\angle 1$ and $\angle 2$ are supplementary.
Practice

Name an angle or angles in the diagram described by each of the following.

1. Supplementary to $\angle AOD$
2. Adjacent and congruent to $\angle AOE$
3. Supplementary to $\angle EOA$
4. Complementary to $\angle EOD$
5. A pair of vertical angles
Vocabulary

Finding Information from a Diagram:

- There are some relationships you can assume to be true from a diagram that has no marks or measures. There are other relationships you cannot assume directly. For example, you can conclude the following from an unmarked diagram.
  - Angles are adjacent.
  - Angles are adjacent and supplementary.
  - Angles are vertical angles.

- You cannot conclude the following from an unmarked diagram.
  - Angles or segments are congruent.
  - An angle is a right angle.
  - Angles are complementary.
Making Conclusions From a Diagram

• What can you conclude from the information in the diagram?

• Can you make each conclusion form the information in the diagram? Explain.
  • $\overline{TW} \cong \overline{WV}$
  • $\overline{PW} \cong \overline{WQ}$
  • $\angle TWQ$ is a right angle
  • $\overline{TV}$ bisects $\overline{PQ}$
Can you make each conclusion from the information in the diagram? Explain.

1. \( \angle J \cong \angle D \)
2. \( \angle JAC \cong \angle DAC \)
3. \( m\angle JCA = m\angle DCA \)
4. \( m\angle JCA + m\angle ACD = 180 \)
5. \( AJ \cong AD \)
6. C is the midpoint of \( JD \)
7. \( \angle JAD \) and \( \angle EAF \) are adjacent and supplementary.
8. \( \angle EAF \) and \( \angle JAD \) are vertical angles.
Vocabulary

• A **linear pair** is a pair of adjacent angles whose noncommon sides are opposite rays. The angles of a linear pair form a straight angle.

• Postulate 1-9 (Linear Pair Postulate): If two angles form a linear pair, then they are supplementary.
Finding Missing Angle Measures

• \( \angle KPL \) and \( \angle JPL \) are a linear pair, \( m\angle KPL = 2x + 24 \), and \( m\angle JPL = 4x + 36 \). What are the measure of \( \angle KPL \) and \( \angle JPL \)?

• \( \angle ADB \) and \( \angle BDC \) are a linear pair. \( m\angle ADB = 3x + 14 \) and \( m\angle BDC = 5x - 2 \). What are \( m\angle ADB \) and \( m\angle BDC \)?
Practice

• Name two pairs of angles that form a linear pair.

• \( \angle EFG \) and \( \angle GFH \) are a linear pair, \( \angle EFG = 2n + 21 \), and \( \angle GFH = 4n + 15 \). What are \( \angle EFG \) and \( \angle GFH \)?
Vocabulary

• An **angle bisector** is a ray that divides an angle into two congruent angles.

• Its endpoint is at the angle vertex.

• Within the ray, a segment with the same endpoint is also an angle bisector.

• The ray or segment bisects the angle.
Using and Angle Bisector to Find Angle Measures

• $\overrightarrow{AC}$ bisects $\angle DAB$. If $m\angle DAC = 58$, what is $m\angle DAB$.

• $\overrightarrow{KM}$ bisects $\angle JKL$. If $m\angle JKL = 72$, what is $m\angle JKM$?
Practice

In the diagram, $\overrightarrow{GH}$ bisects FGI.

A) Solve for $x$ and find $m\angle FGH$.
B) Find $m\angle HGI$.
C) Find $m\angle FGI$
Basic Constructions

Objective: To make basic constructions using a straightedge and a compass.
Objectives

• I can construct congruent segments.
• I can construct congruent angles.
• I can construct the perpendicular bisector.
• I can construct the angle bisector.
Vocabulary

• You can use special geometric tools to make a figure that is congruent to an original figure without measuring. This method is more accurate than sketching and drawing.

• A **straightedge** is a ruler with no markings on it.

• A **compass** is a geometric tool used to draw circles and parts of circles called arcs.

• A **construction** is a geometric figure drawn using a straightedge and a compass.
Constructing Congruent Segments

• Construct a segment congruent to a given segment.
  • Given: $\overline{AB}$
  • Construct: $\overline{CD}$ so that $\overline{CD} \cong \overline{AB}$

• Use a straightedge to draw $\overline{XY}$. Then construct $\overline{RS}$ so that $RS = 2XY$. 
Practice

- Construct $\overline{XY}$ congruent to $\overline{AB}$.
- Construct $\overline{VW}$ so that $VW = 2AB$.
- Construct $\overline{DE}$ so that $DE = TR + PS$.
- Construct $\overline{QJ}$ so that $QJ = TR - PS$. 
Constructing Congruent Angles

• Construct an angle congruent to a given angle.
  • Given: \( \angle A \)
  • Construct \( \angle S \) so that \( \angle S \cong \angle A \)

• Construct \( \angle F \) so that \( m\angle F = 2m\angle B \).
Practice

• Construct $\angle D$ so that $\angle D \cong \angle C$.

• Construct $\angle F$ so that $m\angle F = 2m\angle C$
Vocabulary

• **Perpendicular lines** are two lines that intersect to form right angles. The symbol $\perp$ means “is perpendicular to.”

• A **perpendicular bisector** of a segment is a line, segment, or ray that is perpendicular to the segment at its midpoint.

• The perpendicular bisector bisects the segment into two congruent segments.
Constructing the Perpendicular Bisector

• Construct the perpendicular bisector of a segment.
  • Given: \(AB\)
  • Construct: \(XY\) so that \(XY\) is the perpendicular bisector of \(AB\)

\[ \text{Diagram: A} - \text{B} \]

• Draw \(ST\). Construct its perpendicular bisector.
Practice

• Construct the perpendicular bisector of $AB$.

• Construct the perpendicular bisector of $TR$. 
Constructing the Angle Bisector

• Construct the bisector of an angle.
  • Given: $\angle A$
  • Construct: $\overline{AD}$, the bisector of $\angle A$

• Draw obtuse $\angle XYZ$. Then construct its bisector $\overline{YP}$. 
Practice

• Draw acute $\angle PQR$. Then construct its bisector.

• Draw obtuse $\angle XQZ$. Then construct its bisector.
Midpoint and Distance in the Coordinate Plane

Objective: To find the midpoint of a segment. To find the distance between two points in the coordinate plane.
Objectives

• I can find the midpoint.
• I can find the endpoint.
• I can find distance.
• You can use formulas to find the midpoint and length of any segment in the coordinate plane.

• Midpoint Formulas:

<table>
<thead>
<tr>
<th>Description</th>
<th>Formula</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>On a Number Line: The coordinate of the midpoint is the average or mean of the coordinates of the endpoints.</td>
<td>The coordinate of the midpoint $M$ of $AB$ is $\frac{a+b}{2}$.</td>
<td></td>
</tr>
<tr>
<td>In the Coordinate Plane: The coordinates of the midpoint are the average of the x-coordinates and the average of the y-coordinates of the endpoints.</td>
<td>Given $AB$ where $A(X_1, Y_1)$ and $B(X_2, Y_2)$, the coordinates of the midpoint $AB$ are $M\left(\frac{X_1+X_2}{2}, \frac{Y_1+Y_2}{2}\right)$.</td>
<td></td>
</tr>
</tbody>
</table>
Finding the Midpoint

• \( \overline{AB} \) has endpoints at \(-4\) and \(9\). What is the coordinate of its midpoint?

• \( \overline{EF} \) has endpoints \(E(7, 5)\) and \(F(2, -4)\). What are the coordinates of its midpoint \(M\)?

• \( \overline{JK} \) has endpoints at \(-12\) and \(4\) on a number line. What is the coordinate of its midpoint?

• What is the midpoint of \( \overline{RS} \) with endpoints \(R(5, -10)\) and \(S(3, 6)\)?
Practice

Find the coordinate of the midpoint of the segment with the given endpoints.

1) 2 and 4
2) −9 and 6
3) 2 and −5
4) −8 and −12

Find the coordinates of the midpoint of $\overline{HX}$.

1) $H(0,0) \& X(8,4)$
2) $H(−1,3) \& X(7,−1)$
3) $H(−6.3,5.2) \& X(1.8,−1)$
4) $H(5\frac{1}{2},−4\frac{3}{4}) \& X(2\frac{1}{4},−1\frac{1}{4})$
Vocabulary

• When you know the midpoint and an endpoint of a segment, you can use the Midpoint Formula to find the other endpoint.
Finding and Endpoint

• The midpoint of $\overline{CD}$ is $M(-2, 1)$. One endpoint is $C(-5, 7)$. What are the coordinates of the other endpoint $D$?

• The midpoint of $\overline{AB}$ has coordinates $(4, -9)$. Endpoint $A$ has coordinates $(-3, -5)$. What are the coordinates of $B$?
Practice

The coordinates of point T are given. The midpoint of $\overline{ST}$ is (5, –8). Find the coordinates of point S.

1. $T(0, 4)$
2. $T(5, –15)$
3. $T(10, 18)$
4. $T(–2, 8)$
5. $T(1, 12)$
6. $T(4.5, –2.5)$
Vocabulary

• In a previous lesson, you have learned how to find the distance between two points on a number line. To find the distance between two points in a coordinate plane, you can use the Distance Formula.

• Distance Formula: The distance between two points $A(X_1, Y_1)$ and $B(X_2, Y_2)$ is $d = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$.

• The Distance Formula is based on the Pythagorean Theorem. When you use the Distance Formula, you are really finding the length of a right triangle.
Finding Distance

• What is the distance between $U(-7, 5)$ and $V(4, -3)$? Round to the nearest tenth.

• $\overline{SR}$ has endpoints $S(-2, 14)$ and $R(3, -1)$. What is $SR$ to the nearest tenth?
Find the distance between each pair of points. If necessary, round to the nearest tenth.

1. $J(2, -1) \& K(2, 5)$
2. $A(0, 3) \& B(0, 12)$
3. $Q(12, -12) \& T(5, 12)$
4. $L(10, 14) \& M(-8, 14)$
5. $C(12, 6) \& D(-8, 18)$
6. $R(0, 5) \& S(12, 3)$
7. $N(-1, -11) \& P(-1, -3)$
8. $E(6, -2) \& F(-2, 4)$
9. $X(-3, -4) \& Y(5, 5)$
Finding Distance

• On a zip-line course, you are harnessed to a cable that travels through the treetops. You start at Platform A and zip to each of the other platforms. How far do you travel from Platform B to Platform C? Each grid unit represents 5m.

• How far do you travel from Platform D to Platform E?

<table>
<thead>
<tr>
<th>Platform</th>
<th>Map Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(–45, 15)</td>
</tr>
<tr>
<td>B</td>
<td>(–30, –20)</td>
</tr>
<tr>
<td>C</td>
<td>(–15, 10)</td>
</tr>
<tr>
<td>D</td>
<td>(20, 20)</td>
</tr>
<tr>
<td>E</td>
<td>(30, –15)</td>
</tr>
<tr>
<td>F</td>
<td>(45, 10)</td>
</tr>
</tbody>
</table>
Practice

Use the table. Find the distance between the cities to the nearest tenth.

- Augusta and Brookline
- Brookline and Charleston
- Brookline and Davenport
- Everett and Fairfield

<table>
<thead>
<tr>
<th>City</th>
<th>Map Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augusta</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>Brookline</td>
<td>(8, 2)</td>
</tr>
<tr>
<td>Charleston</td>
<td>(4, 5)</td>
</tr>
<tr>
<td>Davenport</td>
<td>(5, 10)</td>
</tr>
<tr>
<td>Everett</td>
<td>(−2, 5)</td>
</tr>
<tr>
<td>Fairfield</td>
<td>(−8, −3)</td>
</tr>
</tbody>
</table>
Classifying Polygons
Vocabulary

• In geometry, a figure that lies in a plane is called a *plane figure*.

• A **polygon** is a closed plane figure formed by three or more segments. Each segment intersects exactly two other segments at their endpoints. No two segments with a common endpoint are collinear. Each segment is called a *side*. Each endpoint of a side is a *vertex*.

• To name a polygon, start at any vertex and list all the vertices consecutively in a clockwise direction.
Example 1

• Name the polygon. Then identify its sides and angles.
Vocabulary

• You can classify a polygon by its number of sides. The tables below show the names of some common polygons.

<table>
<thead>
<tr>
<th>Sides</th>
<th>Name</th>
<th>Sides</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Triangle or trigon</td>
<td>9</td>
<td>Nonagon or enneagon</td>
</tr>
<tr>
<td>4</td>
<td>Quadrilateral or tetragon</td>
<td>10</td>
<td>Decagon</td>
</tr>
<tr>
<td>5</td>
<td>Pentagon</td>
<td>11</td>
<td>Hendecagon</td>
</tr>
<tr>
<td>6</td>
<td>Hexagon</td>
<td>12</td>
<td>Dodecagon</td>
</tr>
<tr>
<td>7</td>
<td>Heptagon</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Octagon</td>
<td>n</td>
<td>n-gon</td>
</tr>
</tbody>
</table>
Vocabulary

• You can also classify a polygon as concave or convex, using the diagonals of the polygons. A **diagonal** is a segment that connects two nonconsecutive vertices.

• A **convex polygon** has non diagonal with points outside the polygons.

• A **concave polygon** has at least one diagonal with points outside the polygon.
Example 2

• Classify the polygon by its number of sides. Tell whether the polygon is convex or concave.
Perimeter, Circumference, and Area

Objectives: To find the perimeter or circumference of basic shapes. To find the area of basic shapes.
Objectives

• I can find the perimeter of a rectangle.
• I can find circumference.
• I can find perimeter in the coordinate plane.
• I can find area of a rectangle.
• I can find area of a circle.
• I can find area of an irregular shape.
Vocabulary

• Perimeter and area are two different ways of measuring geometric figures.

• The **perimeter** $P$ of a polygon is the sum of the lengths of its sides.

• The **area** $A$ of a polygon is the number of square units it encloses.

• For figures such as squares, rectangles, triangles, and circles, you can use formulas for perimeter (or circumference $C$ for circles) and area.
## Vocabulary

### Perimeter, Circumference, and Area

<table>
<thead>
<tr>
<th>Shape</th>
<th>Formulas</th>
<th>Diagram</th>
</tr>
</thead>
</table>
| **Square** | side length $s$  
$P = 4s$  
$A = s^2$ | ![Square Diagram](image) |
| **Triangle** | side length $a$, $b$, and $c$, base $b$, and height $h$  
$P = a + b + c$  
$A = \frac{1}{2}bh$ | ![Triangle Diagram](image) |
| **Rectangle** | Base $b$ and height $h$  
$P = 2b + 2h$ or $2(b + h)$  
$A = bh$ | ![Rectangle Diagram](image) |
| **Circle** | Radius $r$ and diameter $d$  
$C = \pi d$, or $C = 2\pi r$  
$A = \pi r^2$ | ![Circle Diagram](image) |
Vocabulary

• The units of measurement for perimeter and circumference include inches, feet, yards, miles, centimeters, and meters.

• When measuring area, use square units such as square inches (∙\text{in}^2), square feet (∙\text{ft}^2), square yards (∙\text{yd}^2), square miles (∙\text{mi}^2), square centimeters (∙\text{cm}^2), and square meters (∙\text{m}^2).
Finding the Perimeter of a Rectangle

- The botany club members are designing a rectangle garden for the courtyard of your school. They plan to place edging on the outside of the path. How much edging material will they need?
Finding the Perimeter of a Rectangle

- You want to frame a picture that is 5 in by 7 in. With a 1-in-wide frame.
  - What is the perimeter of the picture?
  - What is the perimeter of the outside edge of the frame?
Practice

• Find the perimeter of each figure.

• A garden that is 5 feet by 6 feet has a walkway 2 feet wide around it. What is the amount of fencing needed to surround the walkway?
Vocabulary

• You can name a circle with a symbol ⊙. For example, the circle with center A is written ⊙ A.

• The formulas for a circle involve the special number pi (π). Pi is the ratio of any circle’s circumference to its diameter. Since π is an irrational number, \( \pi = 3.1415926 \ldots \), you cannot write it as a terminating decimal. For an approximate answer, you can use 3.14 or \( \frac{22}{7} \) for \( \pi \). You can also use the \( \pi \) key on your calculator to get a rounded decimal for \( \pi \). For an exact answer, leave the result in terms of \( \pi \).
Finding Circumference

• What is the circumference of the circle in the terms of \( \pi \)? What is the circumference of the circle to the nearest tenth?

• What is the circumference of a circle with a radius 24 m in terms of \( \pi \)?

• What is the circumference of a circle with a diameter 24 m to the nearest tenth?
Practice

• Find the circumference of each in terms of $\pi$.

- 15 cm
- 5 ft
- 3.7 in
- $\frac{1}{4}$ m
Finding Perimeter in the Coordinate Plane

- What is the perimeter of ΔEFG?

- Graph quadrilateral JKLM with vertices J(−3, −3), K(1, −3), L (1, 4), and M(−3, 1)? What is the perimeter of JKLM?
Practice

Graph each figure in the coordinate plane. Find each perimeter.

• X(0, 2), Y(4, –1), Z(–2, –1)
• A(–4, –1), B(4, 5), C(4, –2)
• L(0, 1), M(3, 5), N(5, 5), P(5, 1)
• S(–5, 3), T(7, –2), U(7, –6), V(–5, –6)
Vocabulary

• To find area, you should use the same unit for both dimensions.
Finding Area of a Rectangle

• You want to make a rectangle banner similar to the one at the right. The banner shown is \(2 \frac{1}{2}\) feet wide and 5 feet high. To the nearest square yard, how much material do you need?

• You are designing a poster that will be 3 yards wide and 8 feet high. How much paper do you need to make the poster? Give your answer in square feet.
Practice

• Find the area of each rectangle with the given base and height.
  • 4 ft, 4 in
  • 30 in, 4 yd
  • 2 ft 3 in, 6 in
  • 40 cm, 2 m

• What is the area of a section of pavement that is 20 feet wide and 100 yards long? Give your answer in square feet.
Finding Area of a Circle

• What is the area in terms of $\pi$?

• The diameter of a circle is 14 feet.
  • What is the area of the circle in terms of $\pi$?
  • What is the area of the circle using an approximation of $\pi$?
Practice

Find the area of each circle in terms of $\pi$.

- Diameter: 4 \text{ in}
- Diameter: 20 \text{ m}
- Diameter: 0.1 \text{ m}
- Diameter: 6.3 \text{ ft}
Vocabulary

• Postulate 1-10 (Area Addition Postulate): The area of a region is the sum of the areas of its nonoverlapping parts.

• This postulate is useful in finding areas of figures with irregular shapes.
Finding Area of an Irregular Shape

• What is the area?
Practice

Find the area of the figures. All angles are right angles.