Linear Functions
Rate of Change and slope

Objective: To find rates of change from tables. To find slope.
Objectives

• I can find the rate of change using a table.

• I can find the slope of an equation using a graph.

• I can find the slope of an equation using two points.

• I can find the slope of horizontal and vertical lines.
Vocabulary

- You can use ratios to show a relationship between changing quantities, such as vertical and horizontal changes.

- **Rate of change** shows the relationship between two changing quantities. When one quantity depends on the other, the following is true:

  \[ \text{rate of change} = \frac{\text{change in dependent}}{\text{change in independent}} \]
Finding Rate of Change Using a Table

The table shows the distance a band marches over time. Is the rate of change in distance with respect to time constant? What does the rate of change represent?

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Distance (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>260</td>
</tr>
<tr>
<td>2</td>
<td>520</td>
</tr>
<tr>
<td>3</td>
<td>780</td>
</tr>
<tr>
<td>4</td>
<td>1040</td>
</tr>
</tbody>
</table>
Finding Rate of Change Using a Table

Is the rate of change in cost constant with respect to the number of pencils bought?

<table>
<thead>
<tr>
<th>Number of Pencils</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>1.75</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
</tr>
</tbody>
</table>
Determine whether each rate of change is constant. If it is, find the rate of change and explain what it represents.

### Turtle Walking

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>21</td>
</tr>
</tbody>
</table>

### Hotdogs and Buns

<table>
<thead>
<tr>
<th></th>
<th>Hotdogs</th>
<th>Buns</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time (min)</strong></td>
<td><strong>1</strong></td>
<td><strong>1</strong></td>
</tr>
<tr>
<td></td>
<td><strong>2</strong></td>
<td><strong>2</strong></td>
</tr>
<tr>
<td></td>
<td><strong>3</strong></td>
<td><strong>3</strong></td>
</tr>
<tr>
<td></td>
<td><strong>4</strong></td>
<td><strong>4</strong></td>
</tr>
</tbody>
</table>

### Airplane Descent

<table>
<thead>
<tr>
<th></th>
<th>Elevation (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time (min)</strong></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>30,000</td>
</tr>
<tr>
<td>2</td>
<td>29,000</td>
</tr>
<tr>
<td>5</td>
<td>27,500</td>
</tr>
<tr>
<td>12</td>
<td>24,000</td>
</tr>
</tbody>
</table>
Vocabulary

- The relationship between time and distance is linear. When data are linear, the rate of change is constant.

- Slope is the ratio of a vertical change (rise) to the horizontal change (run) between two points on the line. The rate of change is called the slope of the line.

\[
\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}}
\]
Finding Slope Using a Graph

What is the slope of the line?

![Graph showing the calculation of slope using the formula \( m = \frac{\text{Rise}}{\text{Run}} = \frac{2}{3} \).]
Finding Slope Using a Graph

What is the slope of the line?
Finding Slope Using a Graph

What is the slope of the line?
Finding Slope Using a Graph

What is the slope of the line?
Finding Slope Using a Graph

What is the slope of the line?
Practice

What is the slope of the line?
Practice

What is the slope of the line?
Practice

What is the slope of the line?
You can use any two points on a line to find its slope. Use subscripts to distinguish between the two points.

The Slope Formula:

\[ \text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}, \text{where } x_2 - x_1 \neq 0 \]

The x-coordinate you use first in the denominator must belong to the same ordered pair as the y-coordinate you use first in the numerator.
Finding Slope Using Points

What is the slope of the line through the points?

a. (−1,0) and (3,−2)

b. (1,3) and (4,−1)

c. (−1,2) and (2,−3)

d. (3,6) and (4,8)
Practice

Find the slope of the line that passes through each pair of points.

1. (0,0) and (3,3)
2. (1,3) and (5,5)
3. (4,4) and (5,3)
4. (0,–1) and (2,3)
5. (–6,1) and (4,8)
6. (2,–3) and (5,–4)
Finding Slopes of Horizontal and Vertical Lines

• What is the slope of the line?

(4, –3) and (4, 2)
(−1, −3) and (5, −3)
Find the slope of each line.
Types of slope

• A line with a positive slope slants upward from left to right.

• A line with a negative slope slants downward from left to right.

• A line with a slope of 0 is horizontal. \((y = 2)\)

• A line with an undefined slope is vertical. \((x = -4)\)
Have you met Mr. Slope guy?
Practice

Without graphing, tell whether the slope of the line that models each linear relationship is positive, negative, zero, or undefined. Then find the slope.

a. The length of a bus route is 4 miles long on the sixth day and 4 miles long on the seventeenth day.

b. A babysitter earns $9 for 1 hour and $36 for 4 hours.

c. A student earns a 98 on a test for answering one question incorrectly and earns a 90 for answering five questions incorrectly.

d. The total cost, including shipping, for ordering five uniforms is $66. The total cost, including shipping, for ordering nine uniforms is $114.
Practice

State the independent variable and the dependent variable in each linear relationship. Then find the rate of change for each situation.

a. Snow is 0.02 m deep after 1 hour and 0.06 m after 3 hours.

b. The cost of tickets is $36 for three people and $84 for seven people.

c. A car is 200 km from its destination after 1 hour and 80 km from its destination after 3 h.
Practice

Each pair of points lies on a line with the given slope. Find x or y.

1. (2,4) and (x,8); slope = $-2$

2. (4,3) and (5,y); slope = $3$

3. (2,4) and (x,8); slope = $-\frac{1}{2}$

4. (3,y) and (1,9); slope = $-\frac{5}{2}$
Direct Variation

Objective: To write and graph an equation of a direct variation.
Objectives

• I can identify a direct variation.

• I can write a direct variation equation.

• I can graph a direct variation equation.

• I can write a direct variation from a table.
Vocabulary

• If the ratio of two variables is constant, then the variables have a special relationship, known as a direct variation.

• A **direct variation** is a relationship that can be represented by a function in the form $y = kx$, where $k \neq 0$.

• The **constant of variation for a direct variation** $k$ is the coefficient of $x$.

• By dividing each side of $y = kx$ by $x$, you can see that the ratio of the variables is constant: $k = \frac{y}{x}$.

• To determine whether an equation represents a direction variation, solve it for $y$. If you can write the equation in the form $y = kx$, where $k \neq 0$, it represents a direct variation.
Identifying a Direct Variation

Does the equation represent a direct variation? If so, find the constant of variation. (k)

a. $7y = 2x$

b. $3y + 4x = 8$

c. $4x + 5y = 0$

d. $-12x = 6y$
Practice

Determine whether each equation represents a direction variation. If it does, find the constant of variation.

a. \(2y = 5x + 1\)

b. \(y + 8 = -x\)

c. \(8x + 9y = 10\)

d. \(-4 + 7x + 4 = 3y\)

e. \(-18x = 6y\)

f. \(0.7x - 1.4y = 0\)
Vocabulary

To write an equation for a direct variation, first find the constant of variation $k$ using an ordered pair, other than $(0,0)$, that you know is a solution of the equation.
Writing a Direct Variation Equation

a. Suppose $y$ varies directly with $x$, and $y = 35$ when $x = 5$. What direct variation equation relates $x$ and $y$? What is the value of $y$ when $x = 9$?

b. Suppose $y$ varies directly with $x$, and $y = 10$ when $x = -2$. What direct variation equation relates $x$ and $y$? What is the value of $y$ when $x = -15$?
Suppose $y$ varies directly with $x$. Write a direct variation equation that relates $x$ and $y$. Then find the value of $y$ when $x = 12$.

a. $y = -10$ when $x = 2$

b. $y = 125$ when $x = -5$

c. $y = 10.4$ when $x = 4$

d. $y = 5$ when $x = 2$

e. $y = 7 \frac{1}{2}$ when $x = 3$

f. $y = 9 \frac{1}{3}$ when $x = -\frac{1}{2}$
Graphing a Direct Variation

Weight on Mars $y$ varies directly with weight on Earth $x$. The weights of the science instruments onboard the Phoenix Mars Lander on Earth and Mars are shown.

- Weight on Mars: 50 pounds
- Weight on Earth: 130 pounds

a. What is an equation that relates weight, in pounds, on Earth $x$ and weight on Mars $y$?

a. What is the graph of the equation?
Graphing a Direct Variation

a. Weight on the moon $y$ varies directly with weight on Earth $x$. A person who weighs 100 pounds on earth weighs 16.6 pounds on the moon. What is an equation that relates weight on Earth $x$ and weight on the moon $y$?

b. What is the graph of this equation?
Practice

Graph each direct variation equation.

a. \( y = 2x \)

b. \( y = -x \)

c. \( y = \frac{1}{3}x \)

d. \( y = -\frac{1}{2}x \)
Practice

a. The difference $d$ you bike varies directly with the amount of time $t$ you bike. Suppose you bike 13.2 miles in 1.25 hours. What is an equation that relates $d$ and $t$?

b. The perimeter $p$ of a regular hexagon varies directly with the length $l$ of one side of the hexagon. What is an equation that relates $p$ and $l$?
Graphs of Direct Variation

The graph of a direct variation equation $y = kx$ is a line with the following properties:

- The line passes through the point $(0,0)$.
- The slope of the line is $k$. 

# Writing a Direct Variation From a Table

For the data in the table, does $y$ vary directly with $x$? If it does, write an equation for the direct variation.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>3.2</td>
</tr>
<tr>
<td>1</td>
<td>2.4</td>
</tr>
<tr>
<td>4</td>
<td>1.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>2.25</td>
</tr>
<tr>
<td>1</td>
<td>-0.75</td>
</tr>
<tr>
<td>4</td>
<td>-3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>11</td>
</tr>
</tbody>
</table>
Practice

• For the data in each table, tell whether \( y \) varies directly with \( x \). If it does, write an equation for the direct variation.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>−6</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>−1.5</td>
</tr>
<tr>
<td>8</td>
<td>−12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5.4</td>
</tr>
<tr>
<td>7</td>
<td>12.6</td>
</tr>
<tr>
<td>12</td>
<td>21.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>11</td>
</tr>
</tbody>
</table>
Objective: To write linear equations using slope-intercept form. To graph linear equations in slope-intercept form.
Objectives

• I can identify slope and y–intercept.

• I can write an equation in slope–intercept form using slope and y-intercept.

• I can write an equation from a graph.

• I can write an equation from two points.

• I can graph a linear equation.

• I can model a function using real world problems.
Vocabulary

• A family of functions is a group of functions with common characteristics.

• A **parent function** is the simplest function with these characteristics.

• The **linear parent function** is \( y = x \) or \( f(x) = x \).

• The graphs of 4 functions are shown at the right.
Vocabulary

• A **linear equation** is an equation that models a linear function.

• In a linear function, the variables cannot be raised to a power other than 1.

• So \( y = 2x \) is a linear equation, but \( y = x^2 \) and \( y = 2^x \) are not linear equations.
Vocabulary

• Graphs of linear functions may cross the y-axis at any point.

• A y-intercept of a graph is the y-coordinate of a point where the graph crosses the y-axis.

• You can use the slope and y–intercept of a line to write and graph an equation of the line.

• Slope – Intercept Form of a Linear Equation:
  • The slope-intercept form of a linear equation of a nonvertical line is \( y = mx + b \).
  • \( m \) = slope
  • \( b \) = y–intercept
Identifying Slope and Y–Intercept

What are the slope and y-intercept of the graph?

a. $y = 5x - 2$

b. $y = -3x + 2$

c. $y = -5x$

d. $y = 4$
Practice

Find the slope and y–intercept of the graph of each equation.

1. \( y = 3x + 1 \)
2. \( y = 5x - 3 \)
3. \( y = -6x \)
4. \( y = 2x - 5 \)
5. \( y = -x + 4 \)
Writing an Equation in Slope–Intercept Form

What is an equation of the line?

a. Slope $-\frac{4}{5}$ and y-intercept 7

b. Slope $\frac{3}{2}$ and y-intercept $-1$

c. Slope 3 and y-intercept 2

d. Slope $-0.5$ and y-intercept 1.5
Practice

Write an equation in slope–intercept form of the line with the given slope and y–intercept.

1. $m = -0.5$, $b = 1.5$

2. $m = 1$, $b = -1$

3. $m = 0.7$, $b = -2$

4. $m = 2$, $b = 3$

5. $m = -2$, $b = \frac{8}{5}$
Writing an Equation From a Graph

What is the equation of the line?
Practice

What is the equation of the line?
Writing an Equation From Two Points

What equation in slope–intercept form represents the line that passes through the points?

a. (2,1) and (5,–8)

b. (3,–2) and (1,–3)

c. (3,2) and (5,4)

d. (–2,–2) and (–4,–4)
Practice

What equation in slope-intercept form represent the line that passes through the points:

1. (2,1) and (5,-8)
2. (3,-2) and (1,-3)
3. (-2,-1) and (4,2)
4. (-3,3) and (1,2)
Graphing a Linear Equation

a. Graph the equation $y = x + 5$.

b. Graph the equation $y = -3x - 1$.

c. Graph the equation $y = 7x$. 
Practice

a. Graph the equation $y = -\frac{1}{2}x + 4$.

b. Graph the equation $2y + 4x = 0$.

c. Graph the equation $y = -5$. 
Modeling a Function

Water pressure can be measured in atmospheres (atm). Use the information below to write an equation that models the pressure $y$ at a depth of $x$ meters. What graph models the pressure?

- At 0 meters, the pressure is 1 atm
- The pressure increases by 0.1 atm/m
Modeling a Function

A plumber charges a $65 fee for a repair plus $35 per hour. Write an equation to model the total cost $y$ of a repair that takes $x$ hours. What graph models the total cost?
Practice

a. Suppose you have a $5–off coupon at a fabric store. You buy fabric that costs $7.50 per yard. Write an equation that models the total amount of money $y$ you pay if you buy $x$ yards of fabric.

b. The temperature at sunrise is 65°F. Each hour during the day, the temperature rises 5°F. Write an equation that models the temperature $y$, in degrees Fahrenheit, after $x$ hours during the day.
Point-Slope Form

Objective: To write and graph linear equations using point-slope form.
Objectives

• I can write an equation in point–slope form.

• I can graph using point–slope form.

• I can write an equation in point–slope form using two points.

• I can write an equation using a table.
Vocabulary

• You can use the slope of a line and any point on the line to write and graph an equation of the line. Any two equations for the same line are equivalent.

• Point – Slope Form of a Linear Equation:
  • The **point-slope form** of an equation of a nonvertical line with slope \( m \) and through point \((x_1,y_1)\) is \( y - y_1 = m(x - x_1) \).
    • \( X_1 \) is the \( x \)-coordinate
    • \( Y_1 \) is the \( y \)-coordinate
    • \( M \) is slope
Given a point \((x_1, y_1)\) on a line and the line’s slope \(m\), you can use the definition of slope to derive point-slope form.

\[
\frac{y_2 - y_1}{x_2 - x_1} = m
\]

Use the definition of slope

\[
\frac{y - y_1}{x - x_1} = m
\]

Let \((x, y)\) be any point on the line. Substitute \((x, y)\) for \((x_2, y_2)\)

\[
\frac{y - y_1}{x - x_1} \times (x - x_1) = m(x - x_1)
\]

Multiply each side by \((x - x_1)\).

\[
y - y_1 = m(x - x_1)
\]

Simplify the left side of the equation.
Writing an Equation in Point–Slope Form

What is an equation of the line?

a. passes through (–3,6) and has slope –5

b. passes through (8,–4) and has slope \( \frac{2}{5} \)

c. passes through (4,0) and has slope –1

d. passes through (4,2) and has slope \( \frac{5}{3} \)
Practice

Write an equation in point–slope form of the line that passes through the given point and with the given slope $m$.

a. $(3,-4); m = 6$

b. $(-2,-7); m = \frac{4}{5}$

c. $(4,2); m = -\frac{5}{3}$

d. $(4,0); m = -1$
Graphing Using Point–Slope Form

What is the graph?

a. \( y - 1 = \frac{2}{3}(x - 2) \)

b. \( y + 7 = -\frac{4}{5}(x - 4) \)

c. \( y + 5 = -(x + 2) \)

d. \( y - 1 = -3(x + 2) \)
Practice

Graph each equation.

a. \( y + 3 = 2(x - 1) \)

b. \( y + 5 = -(x + 2) \)

c. \( y - 1 = -3(x + 2) \)

d. \( y - 2 = \frac{4}{9}(x - 3) \)
Vocabulary

• You can write the equation of a line given any two points on the line.

• First use the two points to find the slope.

• Then use the slope and one of the points to write the equation.
Using Two Points to Write an Equation

What is the equation of the line in point-slope form?

a. \((-2,-3)\) and \((1,4)\)

b. \((2,4)\) and \((-3,-6)\)

c. \((-2,-1)\) and \((1,3)\)

d. \((-6,6)\) and \((3,3)\)
Practice

• Write an equation in point – slope form for each line.
Practice

Write an equation in point–slope form of the line that passes through the given points. Then write the equation in slope–intercept form.

a. \((1,4), (-1,1)\)

b. \((2,4), (-3,-6)\)

c. \((-6,6), (3,3)\)
Using a Table to Write an Equation

The table shows the altitude of a hot–air balloon during its linear descent. What equation in slope–intercept form gives the balloon’s altitude at any time? What do the slope and y–intercept represent?

<table>
<thead>
<tr>
<th>Time, x</th>
<th>Altitude, y</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>640</td>
</tr>
<tr>
<td>30</td>
<td>590</td>
</tr>
<tr>
<td>70</td>
<td>490</td>
</tr>
<tr>
<td>90</td>
<td>440</td>
</tr>
</tbody>
</table>

The table shows the number of gallons of water y in a tank after x hours. The relationship is linear. What is an equation in point – slope form that models the data? What does the slope represent?

<table>
<thead>
<tr>
<th>Time, x</th>
<th>Water, y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3320</td>
</tr>
<tr>
<td>3</td>
<td>4570</td>
</tr>
<tr>
<td>5</td>
<td>7070</td>
</tr>
<tr>
<td>8</td>
<td>10820</td>
</tr>
</tbody>
</table>
Practice

Model the data in each table with a linear equation in slope–intercept form. Then tell what the slope and y–intercept represent.

<table>
<thead>
<tr>
<th>Time Painting, x (days)</th>
<th>Volume of Paint, y (gallons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>56</td>
</tr>
<tr>
<td>3</td>
<td>44</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time worked, x (hours)</th>
<th>Wages Earned, y ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.50</td>
</tr>
<tr>
<td>3</td>
<td>25.50</td>
</tr>
<tr>
<td>6</td>
<td>51.00</td>
</tr>
</tbody>
</table>
Standard Form

Objective: to graph linear equations using intercepts. To write linear equations in standard form.
Objectives

• I can find the x– and y–intercepts.

• I can graph a line using the intercepts.

• I can graph horizontal and vertical lines.

• I can transform from point–slope form and slope–intercept form to standard form.

• I can use standard form as a model for real world problems.
Vocabulary

• Recall that the y-intercept is the y-coordinate of a point where a graph crosses the y-axis.

• The x-intercept is the x-coordinate of a point where the graph crosses the x-axis.

• One form of a linear equation, called standard form, allows you to find intercepts quickly. You can use the intercepts to draw the graph.

• Standard Form of a Linear Equation:
  • The **standard form of a linear equation** is \( Ax + By = C \), where \( A, B, \) and \( C \) are real numbers, and \( A \) and \( B \) are not both zero.
Vocabulary

• When finding the x and y–intercepts of an equation, you are simply replacing either x or y with 0.

• When solving for the x–intercept, replace y with 0.
• When solving for the y–intercept, replace x with 0.
Finding x– and y–intercepts

What are the x– and y–intercepts of the graph?

a. \(3x + 4y = 24\)

b. \(5x - 6y = 60\)

c. \(3x + 8y = 12\)
Practice

Find the x– and y–intercepts of the graph of each equation.

1. \( x + y = 9 \)
2. \( x - 2y = 2 \)
3. \( 7x - y = 21 \)
4. \( -5x + 3y = -7.5 \)
5. \( 3x - 5y = -30 \)
Graphing a Line Using Intercepts

What is the graph?

- \( x - 2y = -2 \)
- \( 2x + 5y = 20 \)
- \( 5x + 4y = 20 \)
Practice

Draw a line with the given intercepts.

1. $x$–intercept: 3, $y$–intercept: 5
2. $x$–intercept: $-1$, $y$–intercept: $-4$
3. $x$–intercept: 4, $y$–intercept: $-3$
4. $-2x + y = 8$
5. $6x - 2y = 18$
Vocabulary

• If $A = 0$ in the standard form $Ax + By = C$, then you can write the equation in the form $y = b$, where $b$ is a constant.

• If $B = 0$, you can write the equation in the form $x = a$, where $a$ is a constant.

• The graph of $y = b$ is a horizontal line, and the graph of $x = a$ is a vertical line.
Graphing Horizontal and Vertical Lines

What is the graph of each equation?
1. \( X = 3 \)
2. \( Y = 3 \)
3. \( X = 4 \)
4. \( x = -1 \)
5. \( y = 0 \)
6. \( y = 1 \)
Practice

For each equation, tell whether its graph is a horizontal or a vertical line. Then graph each.

1. \( y = -6 \)
2. \( x = 3 \)
3. \( x = -1.8 \)
4. \( y = 7 \)
Vocabulary

Given an equation in slope–intercept form or point–slope form, you can rewrite the equation in standard form using only integers.
What is the standard form of the equation?

- $y = \frac{1}{3}x + 5$
- $y = -2x + 6$
- $y = -\frac{4}{5}x - 9$
Practice

Write each equation in standard form

1. \( y - 4 = -2(x - 3) \)

2. \( y = \frac{1}{4}x - 2 \)

3. \( y + 2 = \frac{2}{3}(x + 4) \)

4. \( y = 4x + 6 \)
Using Standard Form as a Model

• A media download store sells songs for $1 each and movies for $12 each. You have $60 to spend. Write and graph an equation that describes the items you can purchase. What are three combinations of numbers of songs and movies you can purchase?

• Suppose the store charged $15 for each movie. What equation describes the number of songs and movies you can purchase for $60?
Practice

In a video game, you earn 5 points for each jewel you find. You earn 2 points for each star you find. Write an equation that represents the numbers of jewels and stars you must find to earn 250 points. What are three combinations of jewels and stars you can find that will earn you 250 points?

A store sells T-shirts for $12 each and sweatshirts for $15 each. You plan to spend $120 on T-shirts and sweatshirts. Write an equation that represents this situation. What are three combinations of T-shirts and sweatshirts you can buy for $120?
Linear Equations

You can describe any line using one or more of these forms of a linear equation. Any two equations for the same line are equivalent.

• Slope – Intercept Form
  \[ y = mx + b \]
  \[ y = -\frac{2}{3}x + 6 \]

• Point – Slope Form
  \[ y - y_1 = m(x - x_1) \]
  \[ y - 4 = -\frac{2}{3}(x - 3) \]

• Standard Form
  \[ Ax + By = C \]
  \[ 2x + 3y = 18 \]
Parallel and Perpendicular Lines

Objective: To determine whether lines are parallel, perpendicular, or neither. To write equations of parallel lines and perpendicular lines.
Objectives

• I can write an equation of a parallel line.

• I can classify lines into parallel, perpendicular, or neither.

• I can write an equation of a perpendicular line.

• I can solve real world problems involving parallel and perpendicular lines.
Vocabulary

• Two distinct lines in a coordinate plane either intersect or are parallel.

• **Parallel lines** are lines in the same plane that never intersect.

• You can determine the relationship between two lines by comparing their slopes and y–intercepts.

• You can use the fact that the slopes of parallel lines are the same to write the equation of a line parallel to a given line.
Vocabulary

Slopes of Parallel Lines

• Nonvertical lines are parallel if they have the same slope and different y–intercepts. Vertical lines are parallel if they have different x–intercepts.

• Example:
  • The graphs of \( y = \frac{2}{3} x + 2 \) and \( y = \frac{2}{3} x - 1 \) are lines that have the same slope, \( \frac{2}{3} \), and different y–intercepts. The lines are parallel.
  • The graphs of \( x = 3 \) and \( x = -1 \) are parallel because they have different x–intercepts.
Writing an Equation of a Parallel Line

What equation represents the line in slope–intercept that passes through the point and is parallel to the given equation?

a. (12,5) \( y = \frac{2}{3}x - 1 \)

b. (–3,–1) \( y = 2x + 3 \)

c. (1,3) \( y = 3x + 2 \)
Practice

What equation represents the line in slope–intercept that is passes through the point and is parallel to the given equation?

a. \((2, -2)\) \(y = -x - 2\)

b. \((2, -1)\) \(y = -\frac{3}{2}x + 6\)

c. \((0,0)\) \(y = \frac{2}{3}x + 1\)
Vocabulary

• You can also use slope to determine whether two lines are perpendicular.

• **Perpendicular lines** are lines that intersect to form right angles.
Vocabulary

Slopes of Perpendicular Lines

• Two nonvertical lines are perpendicular if the product of their slopes is -1. A vertical line and a horizontal line are also perpendicular.

• Example
  • The graph of $y = -4x + 1$ has a slope of $-4$. The graph of $y = \frac{1}{4}x - 2$ has a slope of $\frac{1}{4}$. Since $-4 \left( \frac{1}{4} \right) = -1$, the lines are perpendicular.
  • The graph of $y = 4$ is a horizontal line. The graph of $x = 1$ is a vertical line. Since there is a horizontal line and a vertical line, they are perpendicular.
Vocabulary

• Two numbers whose product is $-1$ are **opposite reciprocals**.

• Perpendicular lines have slopes that are opposite signs and reciprocals of each other. This is known as opposite reciprocals.

• To find the opposite reciprocal of $-\frac{3}{4}$, for example, first find the reciprocal, $-\frac{4}{3}$. Then write its opposite, $\frac{4}{3}$. Since $-\frac{3}{4} \times \frac{4}{3} = -1$, $\frac{4}{3}$ is the opposite reciprocal of $-\frac{3}{4}$. 
Classifying Lines

Are the graphs parallel, perpendicular, or neither?

a. $4y = -5x + 12$ and $y = \frac{4}{5}x - 8$

b. $y = \frac{3}{4}x + 7$ and $4x - 3y = 9$

c. $6y = -x + 6$ and $y = -\frac{1}{6}x + 6$
Practice

Are the following parallel, perpendicular, or neither?

a. \( y = x + 11 \) and \( y = -x + 2 \)

b. \( y = -2x + 3 \) and \( 2x + y = 7 \)

c. \( y = 4x - 2 \) and \( -x + 4y = 0 \)
Writing an Equation of a Perpendicular Line

What equation represents the line in slope–intercept that passes through the point and is perpendicular to the given equation?

a. $(2,4) \quad y = \frac{1}{3}x - 1$

a. $(1,8) \quad y = 2x + 1$

b. $(-2,3) \quad y = \frac{1}{2}x - 1$
Practice

What equation represents the line in slope–intercept that is passes through the point and is perpendicular to the given equation?

a. (1,-2)  \( y = 5x + 4 \)

b. (0,0)  \( y = -3x + 2 \)

c. (5,0)  \( y + 1 = 2(x - 3) \)
Solving a Real – World Problem

An architect uses software to design the ceiling of a room. The architect needs to enter an equation that represents a new beam. The new beam will be perpendicular to the existing beam, which is represented by the line. The new beam will pass through the corner represented by the point. What is an equation that represents the new beam?

What equation could the architect enter to represent a second beam whose graph will pass through the corner at (0,10) and be parallel to the existing beam? Give your answer in slope–intercept form.
Practice

A path for a new city park will connect the park entrance to Main Street. The path should be perpendicular to Main Street. What is an equation that represents the path?

A bike path is being planned for the park. The bike path will be parallel to Main Street and will pass through the park entrance. What is an equation of the line that represents the bike path?
Scatter plots and trend lines

Objective: To write an equation of a trend line and of a line of best fit. To use a trend line and a line of best fit to make predictions.
Objectives

• I can make a scatter plot and describe its correlation.

• I can write an equation of a trend line.

• I can find the line of best fit.

• I can identify whether relationships are causal.
Vocabulary

• You can determine whether two sets of numerical data are related by graphing them as ordered pairs. If the two sets of data are related, you may be able to use a line to estimate or predict values.

• A scatter plot is a graph that relates two different sets of data by displaying them as ordered pairs.

• Most scatter plots are in the first quadrant of the coordinate plane because the data are usually positive numbers.
Vocabulary

When $y$ tends to increase as $x$ increases, the two sets of data have a **positive correlation**.
Vocabulary

When \( y \) tends to decrease as \( x \) increases, the two sets of data have a **negative correlation**.
Vocabulary

When x and y are not related, the two sets of data have **no correlation**.
Making a Scatter Plot and Describing Its Correlation

The table shows the altitude of an airplane and the temperature outside the plane. (a) Make a scatter plot of the data. (b) What type of relationship does the scatter plot show?

<table>
<thead>
<tr>
<th>Altitude (m)</th>
<th>Temperature (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>59.0</td>
</tr>
<tr>
<td>500</td>
<td>59.2</td>
</tr>
<tr>
<td>1000</td>
<td>61.3</td>
</tr>
<tr>
<td>1500</td>
<td>55.5</td>
</tr>
<tr>
<td>2000</td>
<td>41.6</td>
</tr>
<tr>
<td>2500</td>
<td>29.8</td>
</tr>
<tr>
<td>3000</td>
<td>29.9</td>
</tr>
<tr>
<td>3500</td>
<td>18.1</td>
</tr>
<tr>
<td>4000</td>
<td>26.2</td>
</tr>
<tr>
<td>4500</td>
<td>12.4</td>
</tr>
<tr>
<td>5000</td>
<td>0.6</td>
</tr>
</tbody>
</table>
Making a Scatter Plot and Describing Its Correlation

Make a scatter plot of the data in the table below. What type of relationship does the scatter plot show?

<table>
<thead>
<tr>
<th>Dollars Spent</th>
<th>Gallons Bought</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2.5</td>
</tr>
<tr>
<td>11</td>
<td>2.8</td>
</tr>
<tr>
<td>9</td>
<td>2.3</td>
</tr>
<tr>
<td>10</td>
<td>2.6</td>
</tr>
<tr>
<td>13</td>
<td>3.3</td>
</tr>
<tr>
<td>5</td>
<td>1.3</td>
</tr>
<tr>
<td>8</td>
<td>2.2</td>
</tr>
<tr>
<td>4</td>
<td>1.1</td>
</tr>
</tbody>
</table>
Practice

For each table, make a scatter plot of the data. Describe the type of correlation the scatter plot shows.

<table>
<thead>
<tr>
<th>Average Price</th>
<th>Number Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>130</td>
</tr>
<tr>
<td>28</td>
<td>112</td>
</tr>
<tr>
<td>36</td>
<td>82</td>
</tr>
<tr>
<td>40</td>
<td>65</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dollars Spent</th>
<th>Gallons Bought</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2.6</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>2.4</td>
</tr>
<tr>
<td>8</td>
<td>2.2</td>
</tr>
<tr>
<td>13</td>
<td>3.5</td>
</tr>
</tbody>
</table>
Vocabulary

- When two sets of data have a positive or negative correlation, you can use a trend line to show the correlation more clearly.

- A **trend line** is a line on a scatter plot, drawn near the points that shows correlation.
Vocabulary

• You can use a trend line to estimate a value between two known data values to predict a value outside the range of known data values.

• **Interpolation** is estimating a value between two known values.

• **Extrapolation** is predicting a value outside the range of known values.
Writing an Equation of a Trend Line

Make a scatter plot of the data at the right. What is the approximate weight of a 7–month–old panda?

<table>
<thead>
<tr>
<th>Age (months)</th>
<th>Weight (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.5</td>
</tr>
<tr>
<td>2</td>
<td>7.6</td>
</tr>
<tr>
<td>3</td>
<td>12.5</td>
</tr>
<tr>
<td>4</td>
<td>17.1</td>
</tr>
<tr>
<td>6</td>
<td>24.3</td>
</tr>
<tr>
<td>8</td>
<td>37.9</td>
</tr>
<tr>
<td>10</td>
<td>49.2</td>
</tr>
<tr>
<td>12</td>
<td>54.9</td>
</tr>
</tbody>
</table>
Writing an Equation of a Trend Line

Make a scatter plot of the data below. Draw a trend line and write its equation. What is the approximate body length of a 7–month–old panda?

<table>
<thead>
<tr>
<th>Age (months)</th>
<th>Body Length (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.0</td>
</tr>
<tr>
<td>2</td>
<td>11.75</td>
</tr>
<tr>
<td>3</td>
<td>15.5</td>
</tr>
<tr>
<td>4</td>
<td>16.7</td>
</tr>
<tr>
<td>5</td>
<td>20.1</td>
</tr>
<tr>
<td>6</td>
<td>22.2</td>
</tr>
<tr>
<td>8</td>
<td>26.5</td>
</tr>
<tr>
<td>9</td>
<td>29.0</td>
</tr>
</tbody>
</table>
Practice

Use the table to answer the questions.

• Make a scatter plot of the data pairs (year, attendance). Draw a trend line and write its equation. Estimate the attendance at the U.S. theme parks in 2005.

• Make a scatter plot of the data pairs (year, revenue). Draw a trend line and write its equation. Predict the revenue at U.S. theme parks in 2012.

<table>
<thead>
<tr>
<th>Year</th>
<th>Attendance (millions)</th>
<th>Revenue (billions of $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>253</td>
<td>5.7</td>
</tr>
<tr>
<td>1992</td>
<td>267</td>
<td>6.5</td>
</tr>
<tr>
<td>1994</td>
<td>267</td>
<td>7.0</td>
</tr>
<tr>
<td>1996</td>
<td>290</td>
<td>7.9</td>
</tr>
<tr>
<td>1998</td>
<td>300</td>
<td>8.7</td>
</tr>
<tr>
<td>2000</td>
<td>317</td>
<td>9.6</td>
</tr>
<tr>
<td>2002</td>
<td>324</td>
<td>9.9</td>
</tr>
<tr>
<td>2004</td>
<td>328</td>
<td>10.8</td>
</tr>
<tr>
<td>2006</td>
<td>335</td>
<td>11.5</td>
</tr>
</tbody>
</table>
Vocabulary

• The trend line that shows the relationship between two sets of data most accurately is called the **line of best fit**.

• A graphing calculator computes the equation of the line of best fit using a method called linear regression.

• The graphing calculator also gives you the **correlation coefficient** \( r \), a number from -1 to 1, that tells you how closely the equation models the data.
Vocabulary

• The nearer $r$ is to 1 or $-1$, the more closely the data cluster around the line of best fit.

• If $r$ is near 1, the data lie close to a line of best fit with positive slope.

• If $r$ is near $-1$, the data lie close to a line of best fit with negative slope.
Finding the Line of Best Fit

Use a graphing calculator to find the equation of the line of best fit for the data in the table. What is the correlation coefficient to three decimal places? Predict the cost of attending in the 2012–2013 academic year. Predict the cost of attending in the 2016–2017 academic year.

<table>
<thead>
<tr>
<th>Academic Year</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000 – 2001</td>
<td>3508</td>
</tr>
<tr>
<td>2001 – 2002</td>
<td>3766</td>
</tr>
<tr>
<td>2002 – 2003</td>
<td>4098</td>
</tr>
<tr>
<td>2003 – 2004</td>
<td>4645</td>
</tr>
<tr>
<td>2004 – 2005</td>
<td>5126</td>
</tr>
<tr>
<td>2005 – 2006</td>
<td>5492</td>
</tr>
<tr>
<td>2006 – 2007</td>
<td>5836</td>
</tr>
</tbody>
</table>


You should have \( y = ax + b \), \( a = \) _____, \( b = \) _____, \( r = \) _____. R is the correlation coefficient. Put the a and b in to write the equation of the line.
Practice

Using a graphing calculator to find the equation of the line of best fit for the data in the table. Find the value of the correlation coefficient $r$ to three decimal places. Then predict the number of movie tickets sold in the U.S. in 2014.

<table>
<thead>
<tr>
<th>Year</th>
<th>Tickets Sold (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>1289</td>
</tr>
<tr>
<td>1999</td>
<td>1311</td>
</tr>
<tr>
<td>2000</td>
<td>1340</td>
</tr>
<tr>
<td>2001</td>
<td>1339</td>
</tr>
<tr>
<td>2002</td>
<td>1406</td>
</tr>
<tr>
<td>2003</td>
<td>1421</td>
</tr>
<tr>
<td>2004</td>
<td>1470</td>
</tr>
<tr>
<td>2005</td>
<td>1415</td>
</tr>
<tr>
<td>2006</td>
<td>1472</td>
</tr>
<tr>
<td>2007</td>
<td>1470</td>
</tr>
</tbody>
</table>
Vocabulary

• **Causation** is when a change in one quantity causes a change in the second quantity.

• A correlation between quantities does not always imply causation.
Identifying Whether Relationships are Causal

In the following situations, is there likely to be a correlation? If so, does the correlation reflect a causal relationship? Explain.

1. The number of loaves of bread bakes and the amount of flour used.

2. The number of mailboxes and the number of firefighters in a city.

3. The cost of a family’s vacation and the size of their house.

4. The time spent exercising and the number of Calories burned.
Practice

In each situation, tell whether a correlation is likely. If it is, tell whether the correction reflects a causal relationship. Explain your reasoning.

a. The amount of time you study for a test and the score you receive.

b. A person’s height and the number of letters in the person’s name.

c. The shoe size and the salary of a teacher.

d. The price of hamburger at a grocery store and the amount of hamburger sold.
Graphing Absolute Value Functions

Objective: To graph an absolute value function. To translate the graph of an absolute value function.
Objectives

• I can describe translations of absolute value equations.

• I can graph a vertical translation.

• I can graph a horizontal translation.

• I can graph a step function.
Vocabulary

• An **absolute value function** has a V-shaped graph that opens up or down.

• The parent function for the family of absolute value functions is $y = |x|$.

• A **translation** is a shift of a graph horizontally, vertically, or both. The result is a graph of the same size and shape, but in a different position.

• You can quickly graph the absolute value equations by shifting the graph of $y = |x|$.
Describing Translations

Below are the graphs of $y = |x|$ and $y = |x| - 2$. How are the graphs related?
Describing Translations

How is the graph related to the graph of $y = |x|$. 

![Graph of $y = |x|$](https://www.embeddedmath.com)
Practice

Describe how each graph is related to the graph of $y = |x|$. 
The graph of $y = |x| + k$ is a translation of $y = |x|$.

Let $k$ be a positive number.

Then $y = |x| + k$ translates the graph of $y = |x|$ up $k$ units, while $y = |x| - k$ translates the graph of $y = |x|$ down $k$ units.
Graphing a Vertical Translation

What is the graph?

a. \( y = |x| + 2 \)

b. \( y = |x| - 7 \)

c. \( y = |x| + 5 \)

d. \( y = |x| - 4 \)
Graph each function by translating $y = |x|$.

1. $y = |x| - 3$
2. $y = |x| + 7$
3. $y = |x| + 3$
4. $y = |x| - 6$
5. $y = |x| + 6$
6. $y = |x| - 2.5$
Vocabulary

• The graphs below show what happens when you graph $y = |x + 2|$ and $y = |x - 2|$.

• For a positive number $h$, $y = |x + h|$ translates the graph of $y = |x|$ left $h$ units, and $y = |x - h|$ translates the graph of $y = |x|$ right $h$ units.
Graphing a Horizontal Translation

What is the graph?

a. \( y = |x + 5| \)
b. \( y = |x - 5| \)
c. \( y = |x + 1.5| \)
d. \( y = |x - 1.5| \)
Practice

Graph each function by translating $y = |x|$.

1. $y = |x - 3|$
2. $y = |x + 3|$
3. $y = |x - 1|$
4. $y = |x + 6|$
5. $y = |x - 7|$
6. $y = |x + 2.5|$
Vocabulary

• The absolute value function is an example of a piecewise function. A **piecewise function** is a function that has different rules for different parts of its domain.

• For example, when $x \geq 0$, $|x| = x$. When $x \leq 0$, $|x| = -x$. 
Vocabulary

• Another example of a piecewise function is a step function. A **step function** is a function that pairs every number in an interval with a single value. The graph of a step function can look like the steps of a staircase.

• Each piece of a graph is a horizontal segment that is missing its right endpoint, indicated by an open circle.
Graphing a Step Function

A school will charter buses so that the student body can attend a football game. Each bus holds a maximum of 60 students. Make a graph that models the relationship between the number of students \( x \) that go to the game by bus and the number of buses \( y \) that are needed.
Graphing a Step Function

Make a graph that models the relationship between the number of students $x$ that go to the game by bus and the number of buses $y$ that are needed if each bus holds a maximum of 50 students.
Practice

The table lists postage for letters weighing as much as 3 oz. You want to mail a letter that weighs 2.7 oz. Graph the step function. How much will you pay in postage?

<table>
<thead>
<tr>
<th>Weight, x</th>
<th>Price, y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; Weight &lt; 1 oz</td>
<td>$0.44</td>
</tr>
<tr>
<td>1 oz &lt; Weight &lt; 2 oz</td>
<td>$0.61</td>
</tr>
<tr>
<td>2 oz &lt; Weight &lt; 3 oz</td>
<td>$0.78</td>
</tr>
</tbody>
</table>