Solving Systems of Equations and Inequalities
Solving Systems by Graphing

Objective: To solve systems of equations by graphing. To analyze special systems
Objectives

• I can solve a system of equations by graphing.

• I can write a system of equations.

• I can identify systems of equations with infinitely many or no solutions.
Vocabulary

• Two or more linear equations form a **system of linear equations**.

• Any ordered pair that makes *ALL* of the equations in a system true is a **solution of a system of linear equations**.

• You can use systems of linear equations to model problems. Systems of equations can be solved in more than one way. One method is to graph each equation and find the intersection point, if one exists.
Solving Systems by Graphing

Solve each system by graphing.

\[ y = x + 2 \quad \text{and} \quad y = 3x - 2 \]

\[ y = 2x \quad \text{and} \quad y = -2x + 8 \]

\[ y = 5 \quad \text{and} \quad x = -3 \]
Practice

Solve each system by graphing. Check your solution.

1. \( y = \frac{1}{2}x + 7 \) and \( y = \frac{3}{2}x + 3 \)
2. \( y = -x + 3 \) and \( y = x + 1 \)
3. \( 4x - y = -1 \) and \( -x + y = x - 5 \)
4. \( y = 6 \) and \( x = -2 \)
Writing a System of Equations

Scientists studied the weights of two alligators over a period of 12 months. The initial weight and growth rate of each alligator are shown below. After how many months did the alligators weigh the same amount?

- Alligator 1 – Initial Weight: 4 lb., Growth Rate: 1.5 lb. per month
- Alligator 2 – Initial Weight: 6 lb., Growth Rate: 1 lb. per month
Writing a System of Equations

One satellite radio service charges $10 per month plus an activation fee of $20. A second service charges $11.00 per month plus an activation fee of $15. In what month was the cost of the service the same?
Practice

- At a local fitness center, members pay a $20 membership fee and $3 for each aerobics class. Nonmembers pay $5 for each aerobics class. For what number of aerobics classes with the cost for members and nonmembers be the same?

- A plant nursery is growing a tree that is 3 ft tall and grows at an average rate of 1 ft per year. Another tree at the nursery is 4 ft tall and grows at an average rate of 0.5 ft per year. After how many years will the trees be the same height?
Vocabulary

• A system of equations that has at least one solution is **consistent**.

• A consistent system can be either *independent* or *dependent*.

• A consistent system that is **independent** has exactly one solution.

• A consistent system that is **dependent** has infinitely many solutions.

• A system of equations that has no solution is **inconsistent**.
Systems with infinitely Many Solutions or No Solution

What is the solution of each system? Use a graph.

• $2y - x = 2$ and $y = \frac{1}{2}x + 1$
• $y = -x - 3$ and $y = -x + 5$
• $y = 3x - 3$ and $3y = 9x - 9$
Practice

Solve each system by graphing. Tell whether the system has one solution, infinitely many solutions, or no solution.

1. \(y = x + 3\) and \(y = x - 1\)
2. \(y - x = 5\) and \(3y = 3x + 15\)
3. \(2y = x - 2\) and \(3y = \frac{3}{2}x - 3\)
4. \(y = 2x - 2\) and \(2y = 4x - 4\)
Concept Summary – Systems of Linear Functions

- One Solution – The lines intersect at one point. The lines have different slopes. The equations are consistent and independent.

- Infinitely Many Solutions – The lines are the same. The lines have the same slope and y-intercept. The equations are consistent and dependent.

- No Solution – The lines are parallel. The lines have the same slope and different y-intercepts. The equations are inconsistent.
Solving Systems using Substitution

Objective: To solve systems of equations using substitution.
Objectives

• I can solve systems of equations using substitution.

• I can solve for a variable and use substitution to solve systems of equations.

• I can use systems of equations in real world problems.

• I can solve systems with infinitely many solutions or no solution.
Vocabulary

• You can solve linear systems by solving one of the equations for one of the variables. Then substitute the expression for the variable into the other equation. This is called the substitution method.

• Systems of equations can be solved in more than one way. When a system has at least one equation that can be solved quickly for a variable, the system can be solved efficiently using substitution.
Using Substitution

What is the solution of the system? Use substitution.

• \( y = 3x \) and \( x + y = -32 \)

• \( y = 2x + 7 \) and \( y = x - 1 \)

• \( 3x + 2y = 23 \) and \( \frac{1}{2}x - 4 = y \)
Practice

Solve each system using substitution. Check your answer.

1. $x + y = 8$ and $y = 3x$
2. $2x + 2y = 38$ and $y = x + 3$
3. $x + 3 = y$ and $3x + 4y = 7$
4. $y = 8 - x$ and $7 = 2 - y$
5. $y = -2x + 6$ and $3y - x + 3 = 0$
6. $3x + 2y = 23$ and $\frac{1}{2}x - 4 = y$
Solving for a Variable and Using Substitution

What is the solution of the system? Solve for a variable and use substitution.

• $3y + 4x = 14$ and $-2x + y = -3$

• $6y + 5x = 8$ and $x + 3y = -7$

• $4y + 3 = 3y + x$ and $2x + 4y = 18$
Practice

• Solve each system using substitution. Check your answer.

1. \( y - 2x = 3 \) and \( 3x - 2y = 5 \)
2. \( 4x = 3y - 2 \) and \( 18 = 3x + y \)
3. \( 2 = 2y - x \) and \( 23 = 5y - 4x \)
4. \( 4y + 3 = 3y + x \) and \( 2x + 4y = 18 \)
5. \( 7x - 2y = 1 \) and \( 2y = x - 1 \)
6. \( 4y - x = 5 + 2y \) and \( 3x + 7y = 24 \)
Using Systems of Equations

• A snack bar sells two sizes of snack packs. A large snack pack is $5, and a small snack pack is $3. In one day, the snack bar sold 60 snack packs for a total of $220. How many small snack packs did the snack bar sell?

• You pay $22 to rent 6 video games. The store charge $4 for new games and $2 for older games. How many new games did you rent?
Practice

• Adult tickets to a play cost $22. Tickets for children cost $15. Tickets for a group of 11 people cost a total of $228. Write and solve a system of equations to find how many children and how many adults were in the group.

• A school is planning a field trip for 142 people. The trip will use six drivers and two types of vehicles: buses and vans. A bus can seat 51 passengers. A van can seat 10 passengers. Write and solve a system of equations to find how many busses and how many vans will be needed.
Vocabulary

• If you get an identity, like $2 = 2$, when you solve a system of equations, then the system has infinitely many solutions.

• If you get a false statement, like $8 = 2$, then the system has no solution.
Systems With Infinitely Many Solutions or No Solution

• How many solutions does each system have?

  • $x = -2y + 4$ and $3.5x + 7y = 14$

  • $y = 3x - 11$ and $y - 3x = -13$

  • $6y + 5x = 8$ and $2.5x + 3y = 4$
Tell whether the system has one solution, infinitely many solutions, or no solution.

1. \( y = \frac{1}{2}x + 3 \) and \( 2y - x = 6 \)
2. \( 6y = -5x + 24 \) and \( 2.5x + 3y = 12 \)
3. \( x = -7y + 34 \) and \( x + 7y = 32 \)
4. \( 5 = \frac{1}{2}x + 3y \) and \( 10 - x = 6y \)
5. \( 17 = 11y + 12x \) and \( 12x + 11y = 14 \)
6. \( 1.5x + 2y = 11 \) and \( 3x + 6y = 22 \)
Objective: To solve systems by adding or subtracting to eliminate a variable.
Objectives

• I can solve a system by adding equations.

• I can solve a system by subtracting equations.

• I can solve a system by multiplying one equation.

• I can solve a system by multiplying both equations.

• I can find the number of solutions for a system of equations.
Vocabulary

• By the Addition and Subtractions Properties of Equality, if \( a = b \) and \( c = d \), then \( a + c = b + d \) and \( a - c = b - d \).

• In the **elimination method**, you use the addition and subtraction properties of equality to add or subtract equations in order to eliminate a variable in a system.

• There is more than one way to solve a system of equations. Some systems are written in a way that makes eliminating a variable a good method to use.
Solving a System by Adding Equation

• What is the solution of the system? Use elimination.

  • $2x + 5y = 17$ and $6x - 5y = -9$
  • $5x - 6y = -32$ and $3x + 6y = 48$
  • $-3x - 3y = 9$ and $3x - 4y = 5$
Practice

Solve each system using elimination.

1. \(3x + 3y = 27\) and \(x - 3y = -11\)
2. \(-x + 5y = 13\) and \(x - y = 15\)
3. \(5x - y = 0\) and \(3x + y = 24\)
Solving a System by Subtracting Equations

• A theater club sells a total of 101 tickets to its first play. A student ticket cost $1. An adult ticket cost $2.50. Total ticket sales are $164. How many student tickets were sold?

• Washing 2 cars and 3 trucks takes 130 minutes. Washing 2 cars and 5 trucks takes 190 minutes. How long does it take to wash each type of vehicle?
Practice

Solve each system using elimination.

1. \( 2x + 4y = 22 \) and \( 2x - 2y = -8 \)

2. \( 4x - 7y = 3 \) and \( x - 7y = -15 \)

3. \( 6x + 5y = 39 \) and \( 3x + 5y = 27 \)
Vocabulary

• In the previous 2 examples, a variable is eliminated because the sum or difference of its coefficients is zero.

• From the Multiplication Property of Equality, you know that you can multiply each side of an equation to get a new equation that is equivalent to the original. That is $a + b = c$ is equivalent to $d(a + b) = dc$ or $da + db = dc$.

• Since this is true, you can eliminate a variable by adding or subtracting, if you first multiply an equation by an appropriate number.

• You can prove that the results are the same simply by substituting the values for the variables in the original equations to show that the equations are true.
Solving a System by Multiplying One Equation

What is the solution of the system? Use elimination and multiply one equation.

• \(-2x + 15y = -32\) \quad \text{and} \quad 7x - 5y = 17

• \(-5x - 2y = 4\) \quad \text{and} \quad 3x + 6y = 6

• \(2x + 3y = 9\) \quad \text{and} \quad x + 5y = 8
Practice

Solve each system using elimination.

1. $2x + 3y = 9 \text{ and } x + 5y = 8$
2. $3x + y = 5 \text{ and } 2x - 2y = -2$
3. $6x + 4y = 42 \text{ and } -3x + 3y = -6$
What is the solution of the system? Use elimination and multiply both equations.

- \(3x + 2y = 1\) and \(4x + 3y = -2\)
- \(4x + 3y = -19\) and \(3x - 2y = -10\)
- \(6x - 3y = 15\) and \(7x + 4y = 10\)
Practice

Solve each system using elimination.

1. $3x + 2y = 17$ and $2x + 5y = 26$
2. $6x - 3y = 15$ and $7x + 4y = 10$
3. $5x - 9y = -43$ and $3x + 8y = 68$
Vocabulary

• Recall that if you get a false statement as you solve a system, then the system has no solution.

• If you get an identity, then the system has infinitely many solutions.
Finding the Number of Solutions

How many solutions does the system have?

- $2x + 6y = 18$ and $x + 3y = 9$
- $-2x + 5y = 7$ and $-2x + 5y = 12$
- $4x - 8y = 15$ and $-5x + 10y = -30$
Practice

• Tell whether the system has one solution, infinitely many solutions, or no solution.

1. $9x + 8y = 15$ and $9x + 8y = 30$
2. $3x + 4y = 24$ and $6x + 8y = 24$
3. $5x - 3y = 10$ and $10x + 6y = 20$
4. $2x - 5y = 17$ and $6x - 15y = 51$
5. $4x - 7y = 15$ and $-8x + 14y = -30$
6. $4x - 8y = 15$ and $-5x + 10y = -30$
Vocabulary

The flowchart below can help you decide which steps to take when solving a system of equations using elimination.

Can I eliminate a variable by adding or subtracting the given equations?  

- Yes, do so.  
- No, can I multiply one of the equations by a number, and then add or subtract the equations?  

- Yes, do so.  
- No, multiply both equations by different numbers. Then add or subtract the equations.
Applications of Linear Systems

Objective: To choose the best method for solving a system of linear equations.
Objectives

• I can find a break-even point in real-world problems.

• I can identify constraints a viable solutions in real-world problems.

• I can solve a wind or current problem.
Choosing a Method for Solving Linear Systems

• Graphing – When you want a visual display of the equations, or when you want to estimate a solution.

• Substitution – When one equation is already solved for one of the variables, or when it is easy to solve for one of the variables.

• Elimination – When the coefficient of one variable are the same or opposites, or when it is not convenient to use graphing or substitution.
Vocabulary

• You can solve systems of linear equations using a graph, the substitution method or the elimination method. The best method to use depends on the forms of the given equations and how precise the solution should be.

• Systems of equations are useful for modeling problems involving mixtures, rates, and break-even points.

• The break-even point for a business is the point at which income equals expenses.
Finding a Break-Even Point

• A fashion designer makes and sells hats. The material for each has costs $5.50. The hats sell for $12.50 each. The designer spends $1400 on advertising. How many hats must the designer sell to break even?

• A puzzle expert wrote a new Sudoku puzzle book. His initial costs are $864. Binding and packaging each book costs $.80. The price of the book is $2. How many copies must be sold to break even?
Practice

• A bicycle store costs $2400 per month to operate. The store pays an average of $60 per bike. The average selling price of each bicycle is $120. How many bicycles must the store sell each month to break even?

• Producing a musical costs $88,000 plus $5900 per performance. One sold-out performance earns $7500 in revenue. If every performance sells out, how many performances are needed to break even?
Vocabulary

• In real-world situations, you need to consider the constraints described in the problem in order to write equations.

• Once you solve an equation, you need to consider the viability of the solution.

• For example, a solution that has a negative number of hours is not a viable solution.
Identifying Constraints and Viable Solutions

• The local zoo is filling two water tanks for the elephant exhibit. One water tank contains 50 gallons of water and is filled at a constant rate of 10 gal/h. The second water tank contains 29 gallons of water and is filled at a constant rate of 3 gal/h. When will the two tanks have the same amount of water?

• The zoo has two other water tanks that are leaking. One tank contains 10 gallons of water and is leaking at a constant rate of 2 gal/h. The second tank contains 6 gallons and is leaking at a constant rate of 4 gal/h. When will the tanks have the same amount of water?
Practice

• You split $1500 between two savings accounts. Account A pays annual 5% interest and Account B pays 4% annual interest. After one year you have earned a total of $69.50 in interest. How much money did you invest in each account?

• A group of scientists studied the effect of a chemical on various strains of bacteria. Strain A started with 6000 cells and decreased at a constant rate of 2000 cells per hour after the chemical was applied. Strain B started with 2000 cells and decreased at a constant rate of 1000 cells per hour after the chemical was applied. When will the strains have the same number?
Vocabulary

When a plane travels from west to east across the United States, the steady west-to-east winds act as tailwinds. This increases the plane’s speed relative to the ground. When a plane travels from east to west, the winds act as headwinds. This decreases the plane’s speed relative to the ground.

From West to East
air speed + wind speed = ground speed

From East to West
air speed – wind speed = ground speed
Solving a Wind or Current Problem

• A traveler files from Charlotte, North Carolina, to Los Angeles, California. At the same time, another traveler fliers from Los Angeles to Charlotte. The air speed of each plane is the same. The ground speed for Charlotte to Los Angeles is 495 mi/h. The ground speed for Los Angeles to Charlotte is 550 mi/h. What is the air speed? What is the wind speed?

• You row upstream at a speed of 2mi/h. You travel the same distance downstream at a speed of 5mi/h. What would be your rowing speed in still water? What is the speed of the current?
Practice

• A traveler is walking on a moving walkway in an airport. The traveler must walk back on the walkway to get a bag he forgot. The traveler’s groundspeed is 2 ft/s against the walkway and 6 ft/s with the walkway. What is the traveler's speed off the walkway? What is the speed of the moving walkway?

• A kayaker paddles upstream from camp to photograph a waterfall and returns. The kayaker’s speed while traveling upstream is 4 mi/h and downstream is 7 mi/h. What is the kayaker’s speed in still water? What is the speed of the current?
Examples

• A dairy owner produces low-fat milk containing 1% fat and whole milk containing 3.5% fat. How many gallons of each type should be combined to make 100 gallons of milk that is 2% fat?

• One antifreeze solution is 20% alcohol. Another antifreeze solution is 12% alcohol. How many litters of each solution should be combined to make 15 liters of antifreeze solution that is 18% alcohol?
Objective: To graph linear inequalities in two variables. To use linear inequalities when modeling real-world situations.
Objectives

• I can identify solutions of a linear inequality.

• I can graph an inequality in two variables.

• I can graph a linear inequality in one variable.

• I can rewrite to graph an inequality.

• I can write an inequality from a graph.
Vocabulary

• A **linear inequality** in two variables, such as $y > x - 3$, can be formed by replacing the equals sign in a linear equation with an inequality symbol.

• A **solution of an inequality** in two variables is an ordered pair that makes the inequality true.

• A linear inequality in two variables has an infinite number of solutions. These solutions can be represented in the coordinate plane as the set of all points on one side of a boundary line.
Identifying Solutions of a Linear Inequality

• Is the ordered pair a solution of $y > x - 3$? (1,2) or (−3,−7)

• Is the ordered pair a solution of $y \leq \frac{2}{3}x + 4$? (3,6)

• Is the ordered pair a solution of $y \leq -2x + 1$? (2,2)
Practice

Determine whether the ordered pair is a solution of the linear inequality.

1. \( y \leq -2x + 1; (2,2) \)
2. \( x < 2; (-1,0) \)
3. \( y \geq 3x - 2; (0,0) \)
4. \( y > x - 1; (0,1) \)
5. \( y \geq -\frac{2}{5}x + 4; (0,0) \)
6. \( 3y > 5x - 12; (-6,1) \)
Vocabulary

• The graph of a linear inequality in two variables consists of all points in the coordinate plane that represents solutions. The graph is a region called a half-plane that is bounded by a line. All points on one side of the boundary line are solutions, while all points on the other side are not solutions.

Each point on a dashed line IS NOT a solution. A dashed line is used for inequalities with > or <.

Each point on a solid line IS a solution. A solid line is used for inequalities with ≥ or ≤.
Graphing an Inequality in Two Variables

What is the graph of the following inequalities.

1. \( y > x - 2 \)
2. \( y \leq \frac{1}{2} x + 1 \)
3. \( y < -4x - 1 \)
4. \( y \geq -x \)
Practice

Graph each linear inequality.

1. $y \leq x - 1$
2. $y \geq 3x - 2$
3. $y > 2x - 6$
4. $y < 5x - 5$
5. $y \leq \frac{1}{2}x - 3$
6. $y > -3x$
Vocabulary

• An inequality in one variable can be graphed on a number line or in the coordinate plane.

• The boundary line will be a horizontal or vertical line.
Graphing a Linear Inequality in One Variable

What is the graph of each inequality in the coordinate plane?

1. $x > -1$
2. $y \geq 2$
3. $x \leq -5$
4. $y < 2$
Practice

Graph each inequality in the coordinate plane.

• $x \leq 4$
• $y \geq -1$
• $x > -2$
• $y < -4$
Vocabulary

• When a linear inequality is solved for $y$, the direction of the inequality symbol determines which side of the boundary line to shade.

• If the symbol is $<$ or $\leq$, shade below the boundary line.
• If the symbol is $>$ or $\geq$, shade above the line.

• Sometimes you must first solve an inequality for $y$ before using the method described about the determine where to shade.
Rewriting to Graph an Inequality

• An interior decorator is going to remodel a kitchen. The wall above the store and the counter is going to be, tile is 12 ft\(^2\) and papered area is 24 ft\(^2\). The owners can spend $420 or less. Write a linear inequality and graph the solutions. What are three possible prices for the wallpaper and tiles?

• For a party, you can spend no more than $12 on nuts. Peanuts cost $2 per pound. Cashews cost $4 per pound. What are three possible combinations of peanuts and cashews you can buy?
Practice

Graph each inequality in the coordinate plane.

1. $-2x + y \geq 3$
2. $x + 3y < 15$
3. $4x - y > 2$
4. $-x + 0.25y \leq -1.75$
Practice

You budget $200 for wooden planks for outdoor furniture. Cedar costs $2.50 per foot and pine costs $1.75 per foot. Let $x$ = the number of feet of cedar and let $y$ = the number of feet of pine. What is an inequality that shows how much of each type of wood can be bought? Graph the inequality. What are three possible amounts of each type of wood that can be bought within your budget?
Writing an Inequality From a Graph
Practice

• Write a linear inequality that represents each graph.
Practice

Write a linear inequality that represents each graph.
Systems of Linear Inequalities

Objective: To solve systems of linear inequalities by graphing. To model real-world situations using systems of linear inequalities.
Objectives

• I can graph a system of inequalities.

• I can write a system of inequalities from a graph.

• I can use a system of inequalities to solve real-world problems.
Vocabulary

• A **system of linear inequalities** is made up of two or more linear inequalities.

• A **solution of a system of linear inequalities** is an ordered pair that makes all the inequalities in the system true.

• The graph of a system of linear inequalities is the set of points that represent all of the solutions of the system.

• You can graph the solutions of a system of linear inequalities in the coordinate plane. The graph of the system is the region where the graphs of the individual inequalities overlap.
Graphing a System of Inequalities

What is the graph of the system?

1. \( y < 2x - 3 \) and \( 2x + y > 2 \)
2. \( y \geq -x + 5 \) and \( -3x + y \leq -4 \)
3. \( y < 2x - 3 \) and \( y > 5 \)
4. \( y \leq 2x - 5 \) and \( x \geq -6 \)
Practice

Solve each system of inequalities by graphing.

1. \( y > 2x + 4 \) and \( y < 3x + 7 \)
2. \( x + 2y \leq 10 \) and \( x + 2y \geq 9 \)
3. \( y > -5x + 2 \) and \( y \geq -3x + 7 \)
4. \( 6x - 5y < 15 \) and \( x + 2y \geq 7 \)
5. \( 2x - \frac{1}{4}y < 1 \) and \( 4x + 8y > 4 \)
Vocabulary

• You can combine your knowledge of linear equations with your knowledge of inequalities to describe a graph using a system of inequalities.
Writing a System of Inequalities From a Graph

What system of inequalities is represented by the graph below?
Writing a System of Inequalities From a Graph

What system of inequalities is represented by the graph below?
Writing a System of Inequalities From a Graph

What system of inequalities is represented by the graph below?
Practice
Practice
Vocabulary

• You can model real-world situations by writing and graphing systems of linear inequalities.

• Some real-world situations involve there or more restrictions, so you must write a system of at least three inequalities.
Using a System of Inequalities

• You are planning what to do after school. You can spend at most 6 hours daily playing basketball and doing homework. You want to spend less than 2 hours playing basketball. You must spend at least 1½ hours on homework. What is a graph showing how you can spend your time?

• You want to build a fence for a rectangular dog run. You want the run to be at least 10 feet wide. The run can be at most 50 feet long. You have 126 feet of fencing. What is a graph showing the possible dimensions of the dog run?
Practice

Suppose you have a job mowing lawns that pays $12 per hour. You also have a job at a clothing store that pays $10 per hour. You need to earn at least $350 per week, but you can work no more than 35 hours per week. You must work a minimum of 10 hours per week at the clothing store. What is a graph showing how many hours per week you can work at each job?